Review of modal testing

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Presentation layout

- Modelling vibration problems
- Aim of modal testing
- Types of modal testing: input-output versus output only
- Types of excitation used in output only and input-output
- Concluding remarks

WHAT IS MODAL TESTING

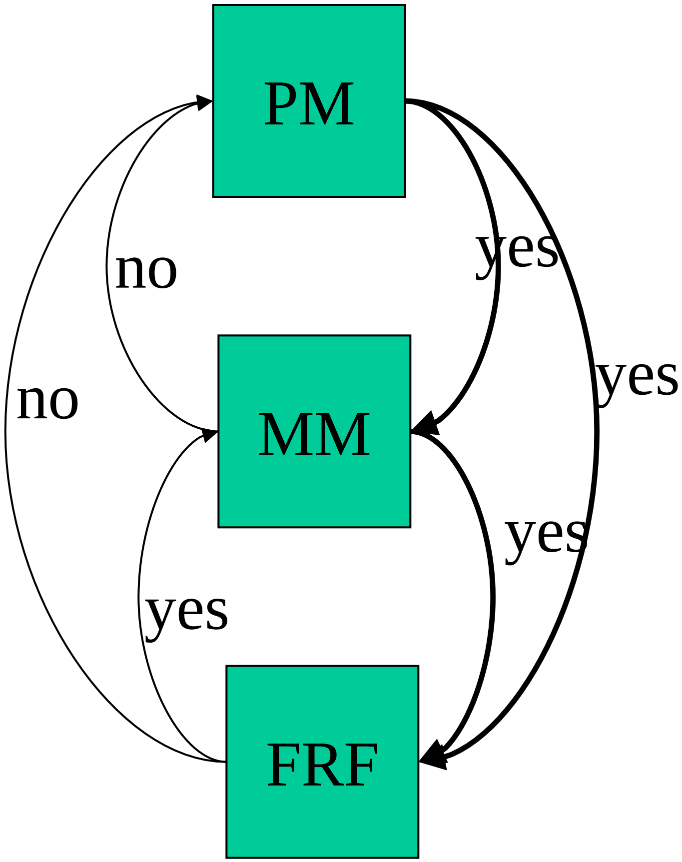
Ewins states: Modal Testing is the process involved in testing components or structures with the objective of obtaining a mathematical description of their dinamic or vibration behavior to:

- Analyze the effect of dynamic loads
- Understand the intrinsic structural dynamics behaviour
- Derive optimal design modifications

The models for vibration description are:

- Physical model **(M, C, K)** PM
- Modal model (eigenvalues Λ, normal modes Φ, modal damping Δ) MM
- Response model (FRF model $H(\omega)$) FRF

MODELLING VIBRATION PROBLEMS



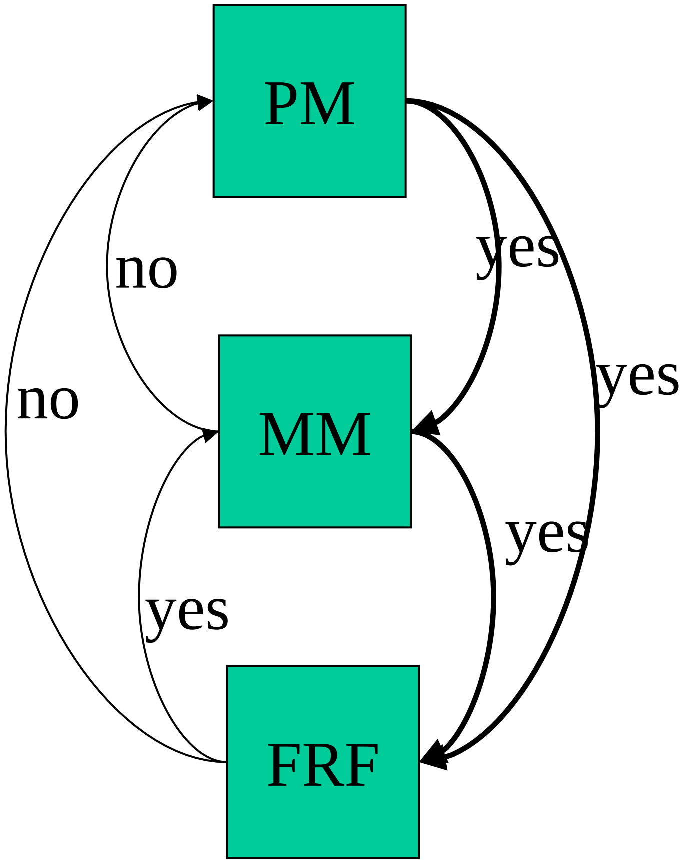
- From PM to MM: by computing eigenvalues and eigenvectors of the problem $(-\lambda M+K)\Phi=0$

- From PM to FRF: $H(\omega)=[-\omega^2 M+j\omega C+K]^{-1}$

- From MM to FRF: $H(\omega)=\Phi\ N\ \Phi^T$
  being N a diagonal matrix = $[k_r - \omega^2 m_r]^{-1}$

- From FRF to MM: by a curve-fitting process (modal parameters estimation, SDOF, MDOF)

MODELLING VIBRATION PROBLEMS (cont'd)



- From MM to PM:

$$\Phi^T M\, \Phi = I \text{ and } \Phi^T M\, \Phi = \Lambda$$

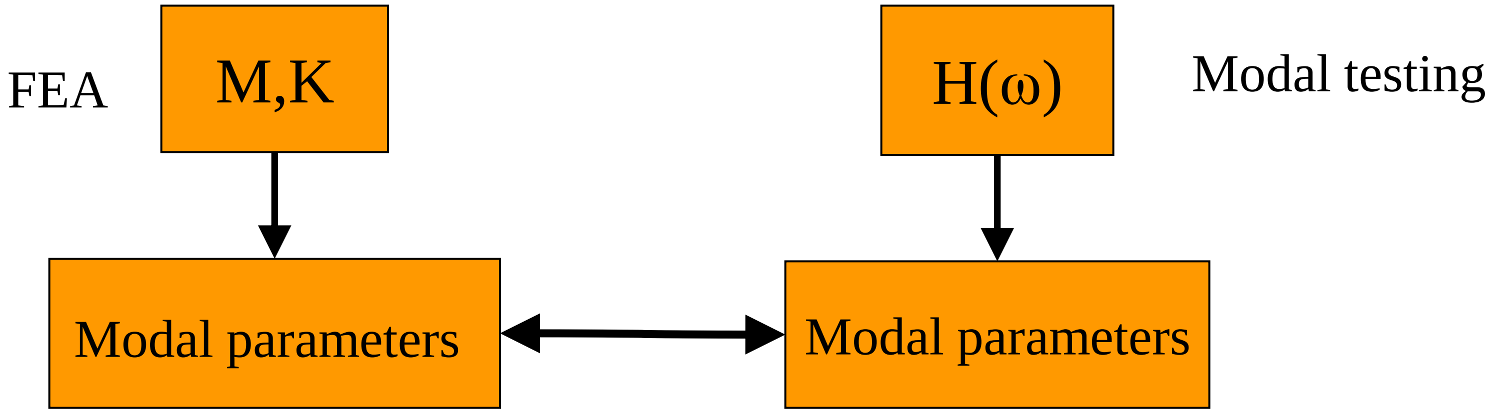
$$\Rightarrow \quad M = (\Phi^T)^{-1}\, \Phi^{-1} \qquad K = (\Phi^T)^{-1}\, \Lambda\, \Phi^{-1}$$

But … <u>only for complete systems</u>. Otherwise not possible.

- From FRF to PM: not possible (more unknowns than equation) using a single frequency. Possible using a set of frequencies, but not reliable

AIM OF MODAL TESTING

- Compare analytical/theoretical/numerical data with experimental data
- Correlate finite element matrices (M, K) by comparing the FRF or modal parameters with those obtained experimentally
- <u>Identify the modal parameters of structures that are difficult to determine from theoretical/numerical models.</u> (These can be used subsequently for numerical computation in modal coordinates).
- Structural modification
- Health monitoring



TYPES OF MODAL TESTING

Refer specifically the modal testing as tests aimed to identify modal parameters. Such tests can be subdivided into two sets:

- input-output tests $\begin{cases} -\text{SISO} \\ -\text{SIMO} \\ -\text{MIMO} \end{cases}$

- output-only tests

The first require measurement of excitation and response: generally used in the lab.
The second, also called environmental test, measures only the response, the excitation being provided by environmental sources, e.g. the wind, traffic, etc. Generally used for large structures (bridges, buildings …)

THEORETICAL BASIS FOR INPUT-OUTPUT

Input – output techniques are developed in the frequency domain and the aim of modal testing is determining the FRF of the considered component or structure.

The relationship permitting to determine the FRF for deterministic excitation is any of the following:

$$\underline{\mathrm{W}}(\omega)=\mathbf{H}(\omega)\,\underline{\mathrm{F}}(\omega) \qquad \underline{\dot{\mathrm{W}}}(\omega)=\mathbf{M}(\omega)\,\underline{\mathrm{F}}(\omega) \qquad \underline{\ddot{\mathrm{W}}}(\omega)=\mathbf{\Gamma}(\omega)\,\underline{\mathrm{F}}(\omega)$$

H = receptance **M** = mobility **Γ** = inertance

<u>*For random excitation it is different (see later)*</u>

Theoretical basis (cont'd)

Specifically for the receptance:

$$\underline{W}(\omega) = \mathbf{H}(\omega)\,\underline{F}(\omega) \quad \Rightarrow \quad w_i = H_{i1} f_1 + H_{i2} f_2 + \ldots + H_{in} f_n$$

Thus

$$H_{ij} = \left. \frac{w_i}{f_j} \right|_{f_k = 0\ (k \neq j)}$$

NOTE: the inverse relationship $\underline{F}(\omega) = \mathbf{Z}(\omega)\,\underline{W}(\omega)$ cannot be used in the experiments because it would imply blocking the displacements. In fact:

$$f_i = Z_{i1} w_1 + Z_{i2} w_2 + \ldots + Z_{in} w_n$$

Theoretical basis (cont'd)

Explicitly

$$\begin{bmatrix} w_1 \\ w_2 \\ \dots \\ w_n \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} \dots & H_{1n} \\ H_{21} & H_{22} & H_{23} \dots & H_{2n} \\ \dots & \dots & \dots & \dots \\ H_{n1} & H_{n2} & H_{n3} \dots & H_{nn} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ \dots \\ f_n \end{bmatrix}$$

To determine the modal parameters only one row or column of the matrix is sufficient (you will see it in one of the next lectures)

Theoretical basis (cont'd)

For random excitation, the relationship permitting to determine the FRF (receptance) is any of the following:

$S_{fw}(\omega) = H_1(\omega)\, S_{ff}(\omega)$ $\quad$ $S_{ww}(\omega) = H_2(\omega)\, S_{wf}(\omega)$

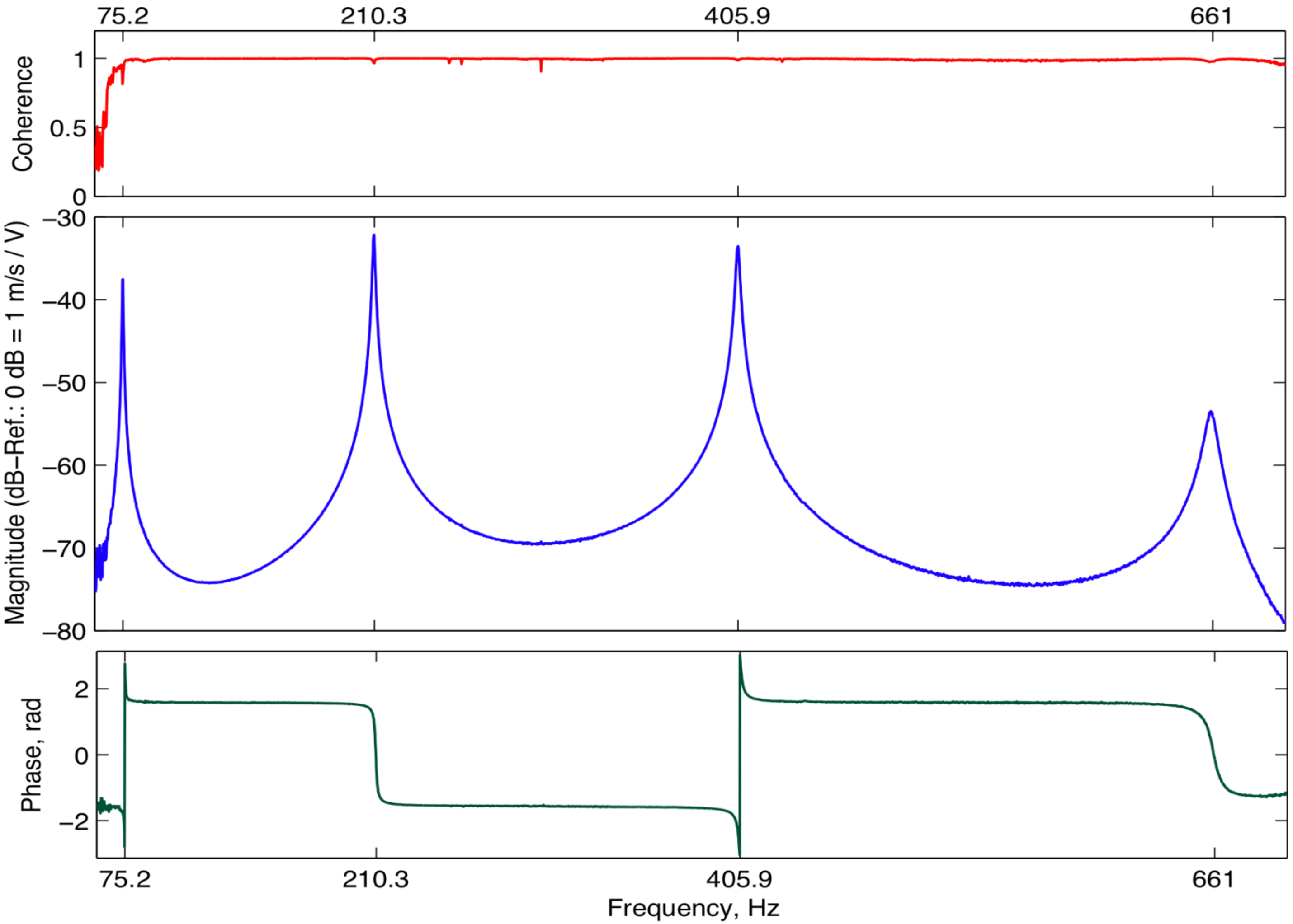
H_2 is a better estimator near resonances, H_1 close to antiresonances.

With random excitation (or repeated impulse excitation) the coherence is used. It gives indication whether noise is affecting measurements.
It is defined as:

$$\gamma^2 = \frac{|S_{fw}(\omega)|^2}{S_{ww} + S_{ff}}$$

γ^2 ranges between 0 and 1. When it is 1, input and output are totally correlated; as it approaches 0, noise affects badly the measurements.

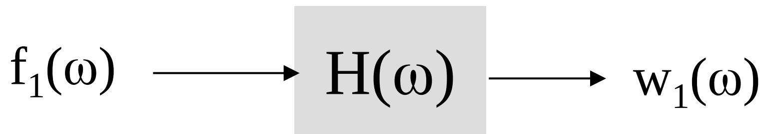
Theoretical basis (cont'd)



Theoretical basis (cont'd)

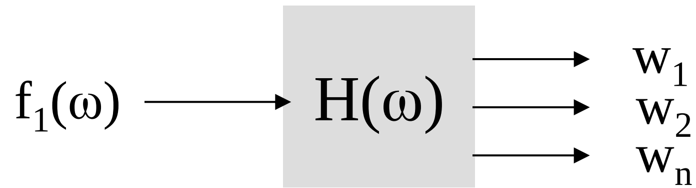
In the SISO test a single input is applied and a single response is measured, so that a single element of the **H** matrix is determined

$$H_{ij} = \left. \frac{w_i}{f_j} \right|_{f_k = 0\ (k \neq j)}$$



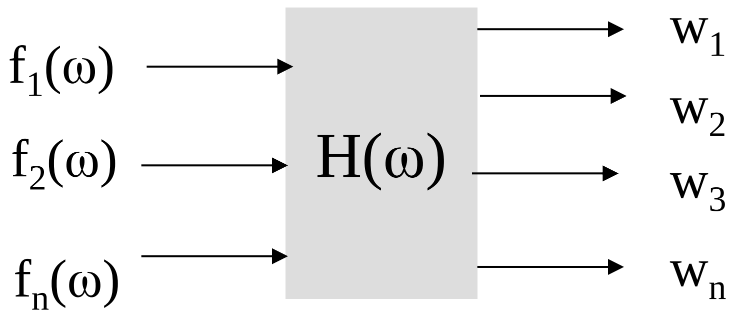
In the SIMO test a single input is applied and a set of different responses are measured, i.e. different elements of a column of the **H** matrix are determined

$$H_{ij} = \left. \frac{w_i}{f_j} \right|_{f_k = 0\ (k \neq j)} \qquad i = 1,2 \ldots n$$



In the MIMO test different inputs are applied and different elements of the matrix are determined.

Theoretical basis (MIMO) (cont'd)



$$\underline{W}(\omega)=\mathbf{H}_{wf}(\omega)\underline{F}(\omega)+\underline{N}(\omega) \quad \Rightarrow \quad \underline{W}^{T}(\omega)=\underline{F}^{T}(\omega)\mathbf{H}_{wf}{}^{T}(\omega)+\underline{N}^{T}(\omega)$$

Pre-multiplying by $\underline{F}^{*}(\omega)$ and taking expected values, being the noise not correlated:

$$\mathbf{S}_{fw}(\omega)=\mathbf{S}_{ff}(\omega)\mathbf{H}_{wf}(\omega)$$

so that a least square estimate of the FRF is determined as:

$$\mathbf{H}_{wf}(\omega)=[\mathbf{S}_{ff}(\omega)^{-1}\mathbf{S}_{fw}(\omega)]^{T}$$

EXPERIMENTAL MODAL ANALYSIS
Ambient Excitation Tests

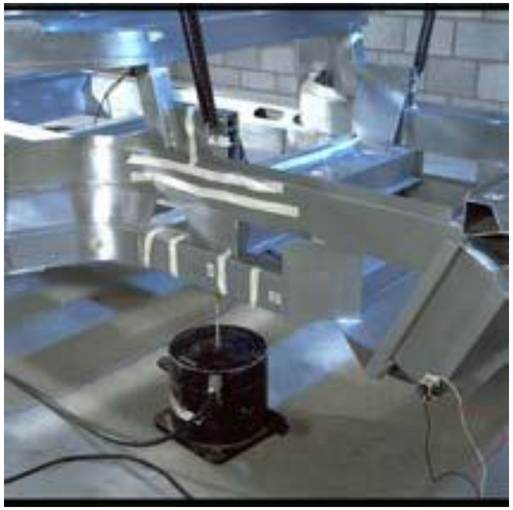
- Many applications do not allow input/output tests
- No possibility to apply input
- Large ambient excitation levels are present
- Specific approach:
  - Use output-only data (responses)
  - Assume white noise excitation

MEASUREMENT TECHNIQUES

The basic measurement technique for input-output consists in:
- an excitation mechanism
- a transduction system
- an analyzer (either PC or appropriate frequency analyzer)

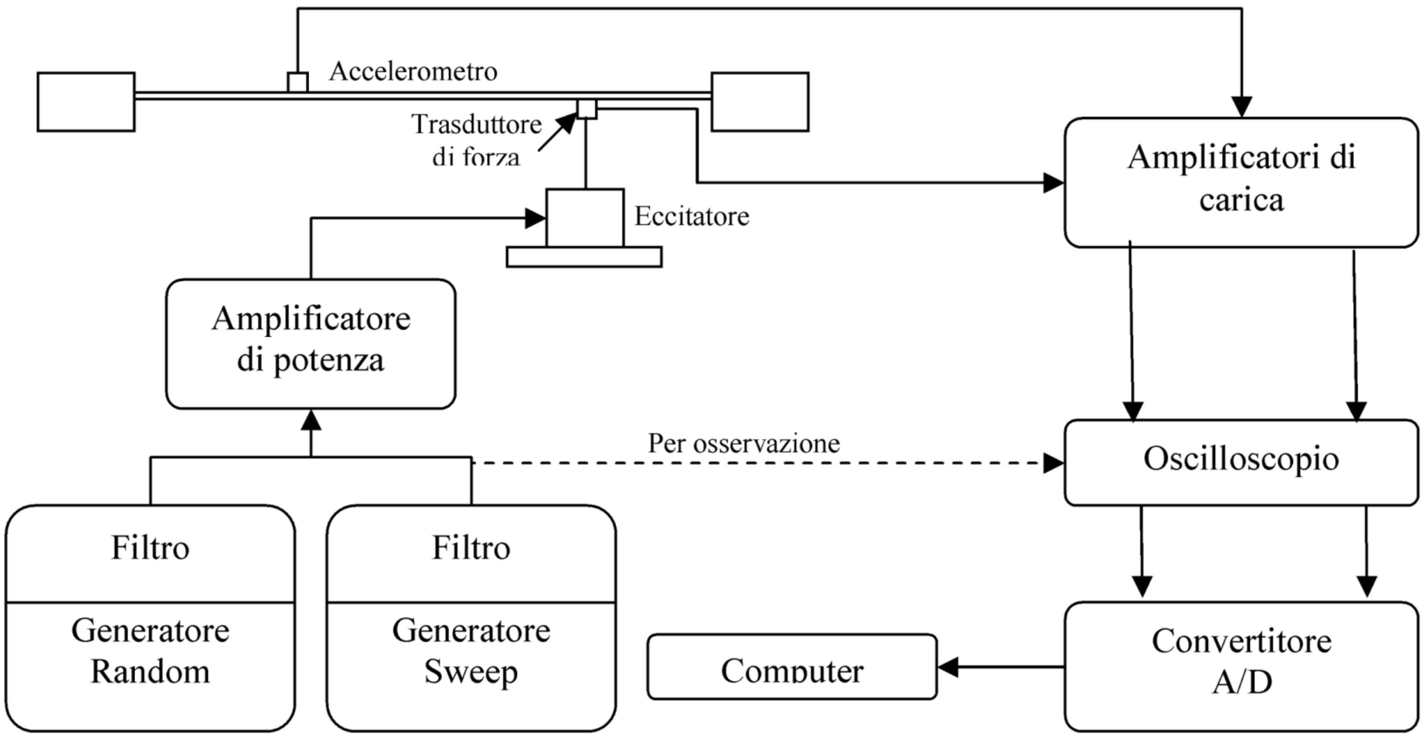
More in detail, the main items in the measurement chain are:
- an exciting source (shaker, hammer, …) moved by a signal generator
- a power amplifier
- a set of transducers with conditioning amplifiers to measure the force applied (force transducer) and response(s) (in general accelerometers or or laser vibrometers)
- a set of appropriate anti-aliasing filters
- an analyzer
- a post-processing software

EXPERIMENTAL MODAL ANALYSIS (EMA): TEST PROCEDURE



MEASUREMENT CHAIN

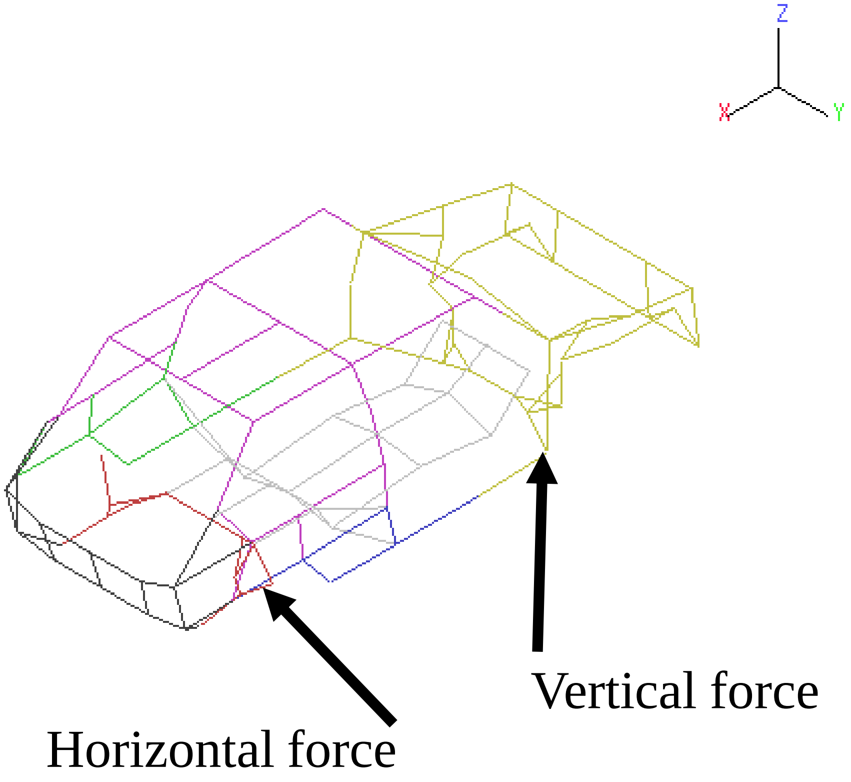
A typical layout of the measurement system is shown in the figure



EXPERIMENTAL MODAL ANALYSIS
A Typical Experiment

- Vehicle Body Test
- **F** : 2 inputs, indicated by arrows
- **X** : 240 outputs: all nodes in picture
- **H** has 480 elements

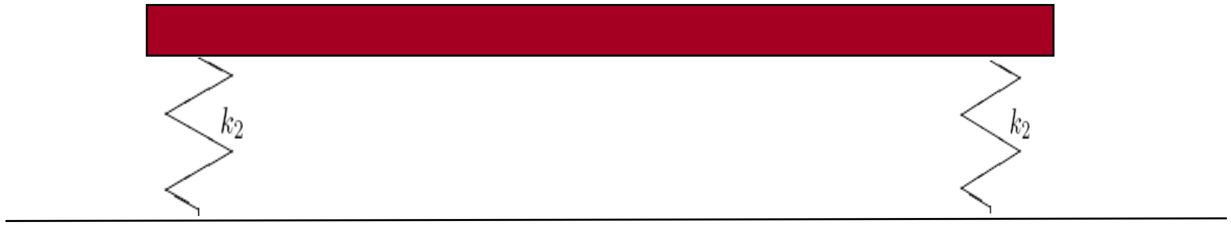
- $\mathbf{X}_{(240x1)} = \mathbf{H}_{(240X2)(}\ \mathbf{F}_{(2x1)}$



FREE VERSUS GROUNDED SUPPORTS

For test of component is situ, the test must be carried out with the real boundary conditions. For laboratory testing the structure can be tested either in free conditions or in grounded conditions

In free conditions the structure is usually suspended by soft springs and thus presents rigid body modes: if the corresponding natural frequencies are quite far away from the elastic natural frequencies, these are not influenced significantly.



In grounded conditions (difficult to achieve in most cases) all the elastic natural frequencies (especially the lowest ones) are influenced by the constraint conditions.

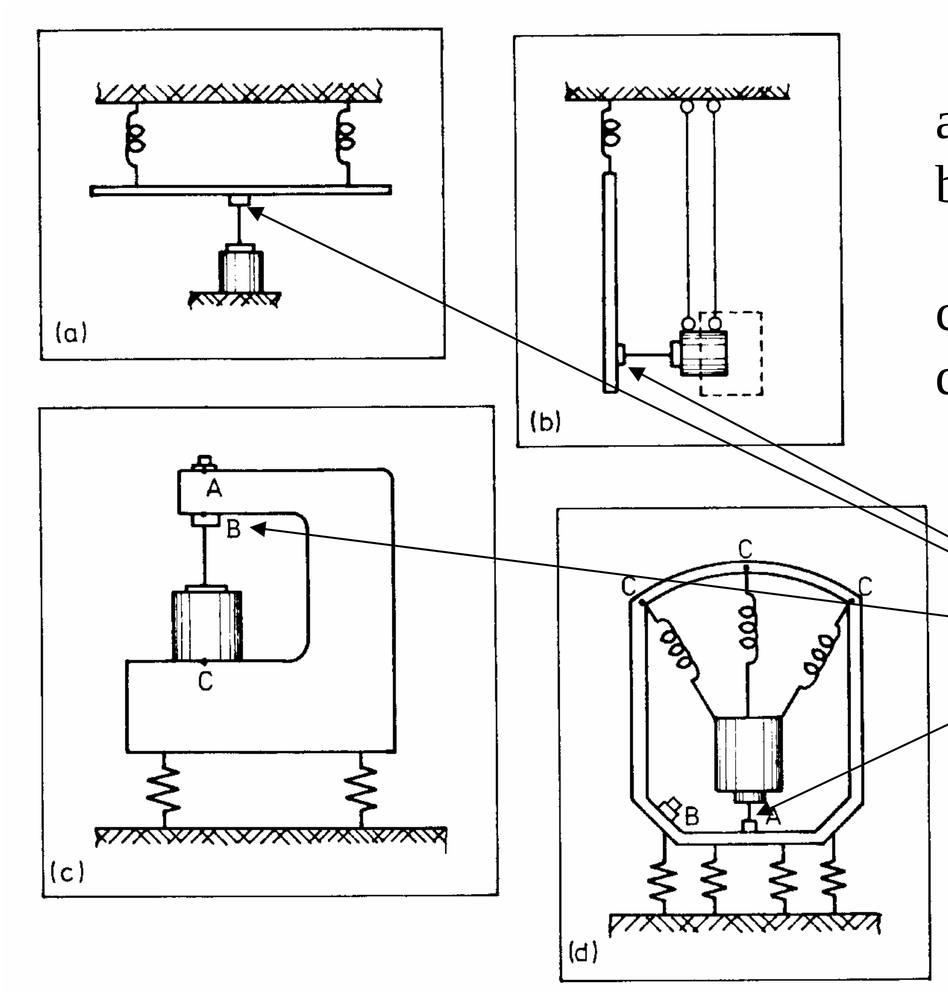
DIFFERENT TYPES OF EXCITATION

The most standard excitations in modal testing are provided by an instrumented hammer or by an electrodynamic shaker

Force transducer

While the hammer is powered by the arm-hand of the operator, the shaker is powered by an appropriate signal generator

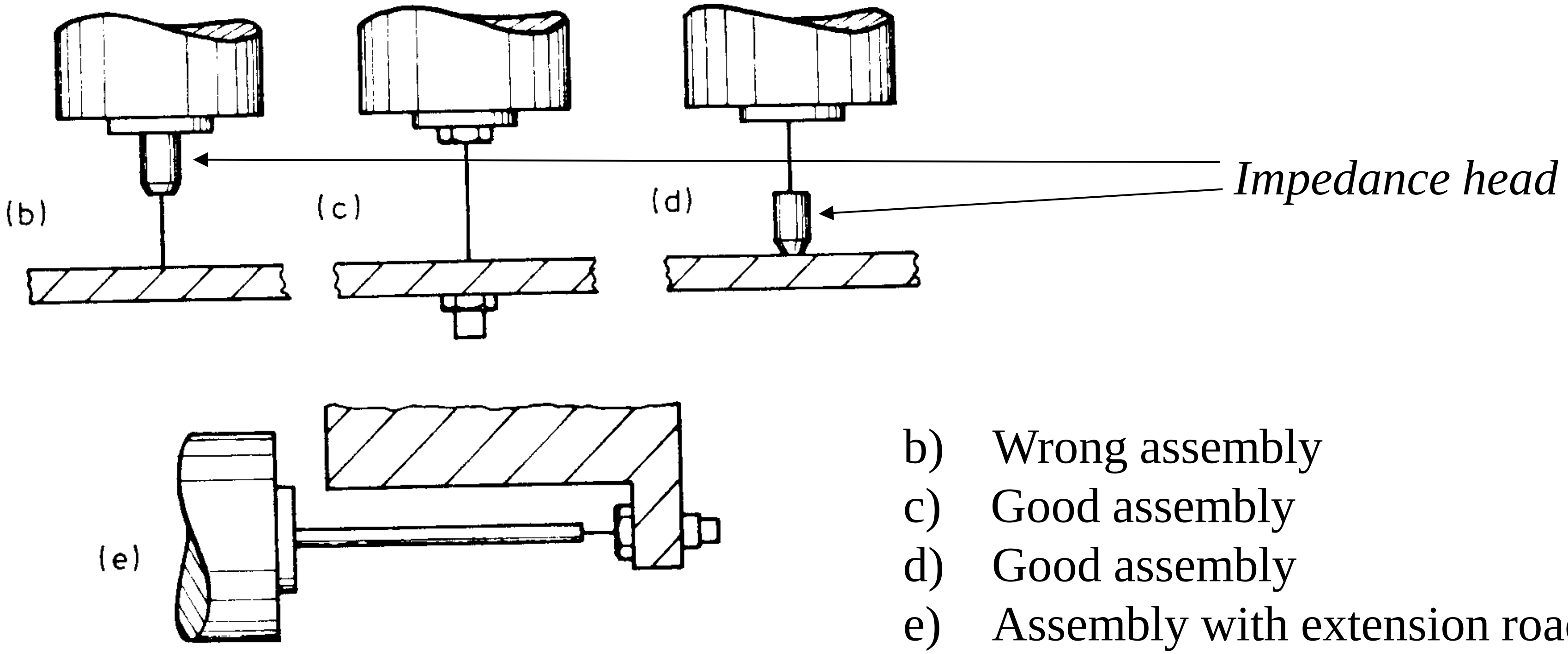
EXCITER ARRANGEMENTS



a) Ideal configuration
b) Exciter suspended with inertia mass
c) Wrong configuration
d) Acceptable compromise

Force transducer

EXCITER ATTACHMENT TO THE TEST STRUCTURE



b) Wrong assembly
c) Good assembly
d) Good assembly
e) Assembly with extension road

DIFFERENT TYPES OF EXCITATION

The excitation source can be any of the following:

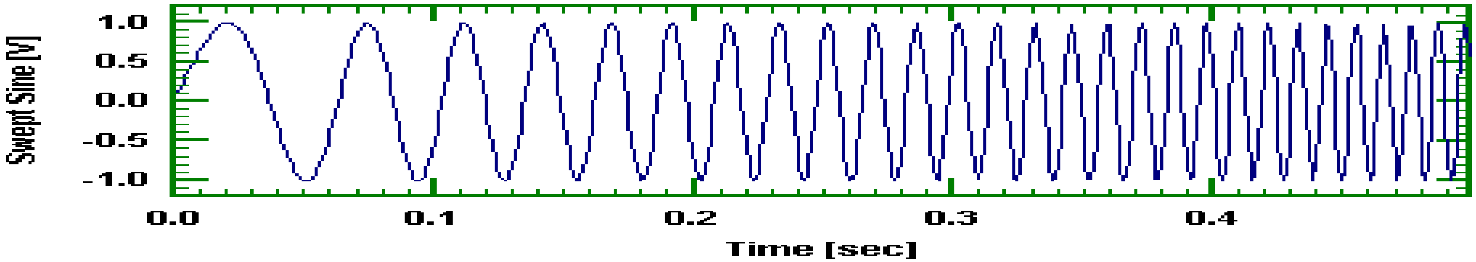
Steady or quasi steady excitations

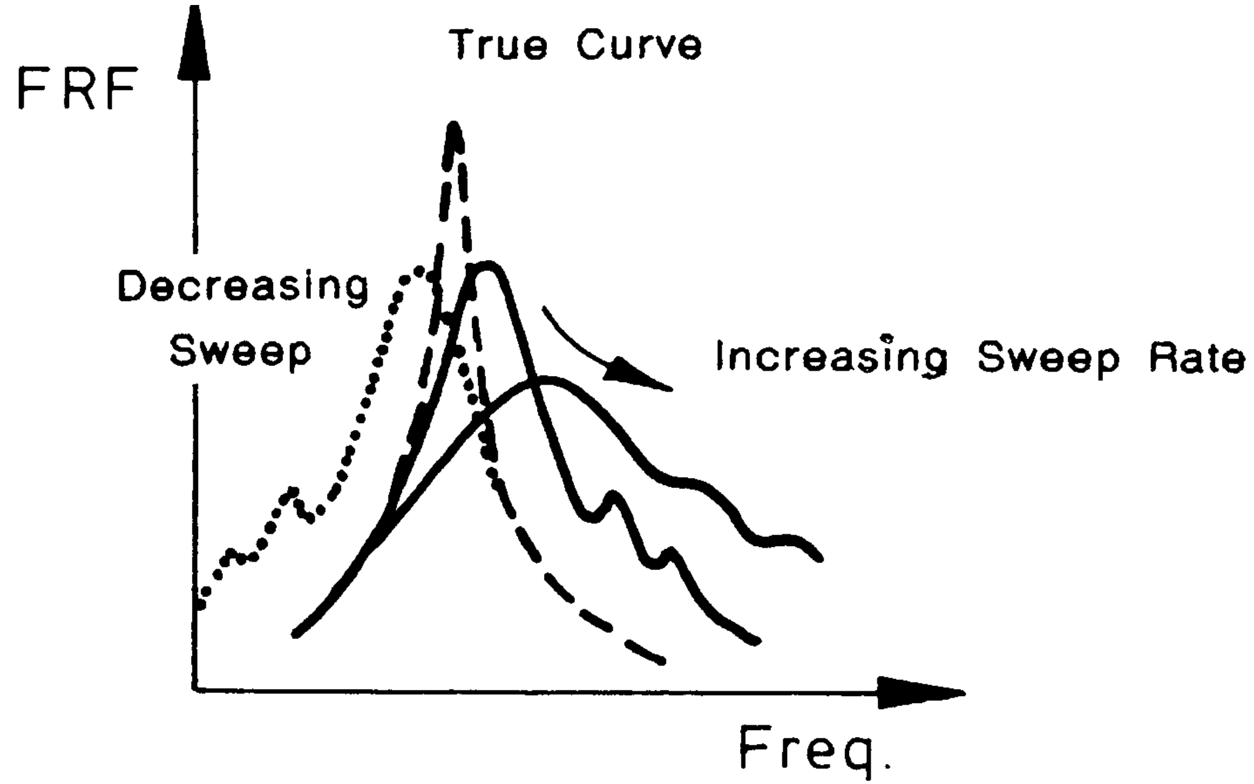
- stepped-sine (provided by the shaker driven by signal generator)
- slow sweep sine (idem)

Transient excitations

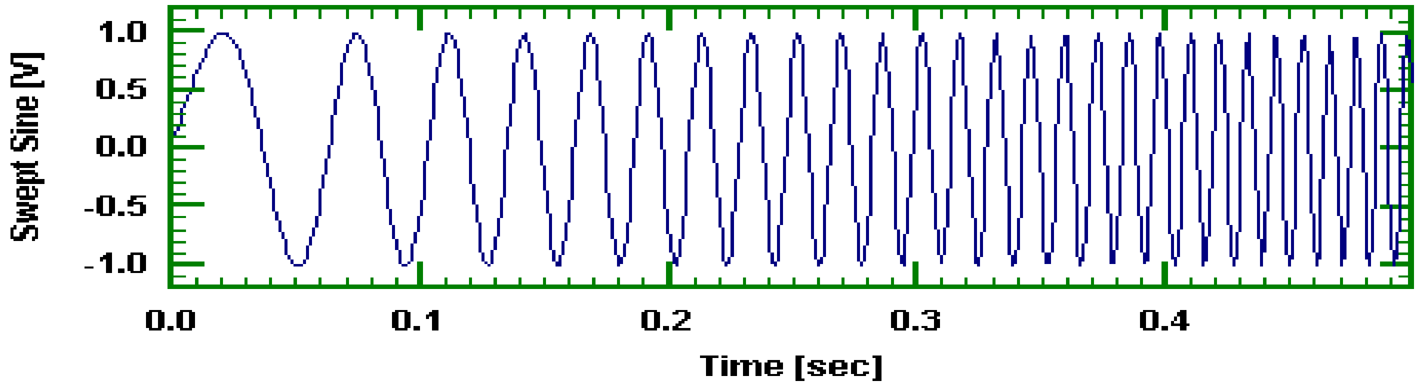
- impulse (provided by a hammer)
- chirp or fast sweep sine (provided by the shaker driven by a suitable signal generator)
- random (provided by a shaker driven by a random signal generator)

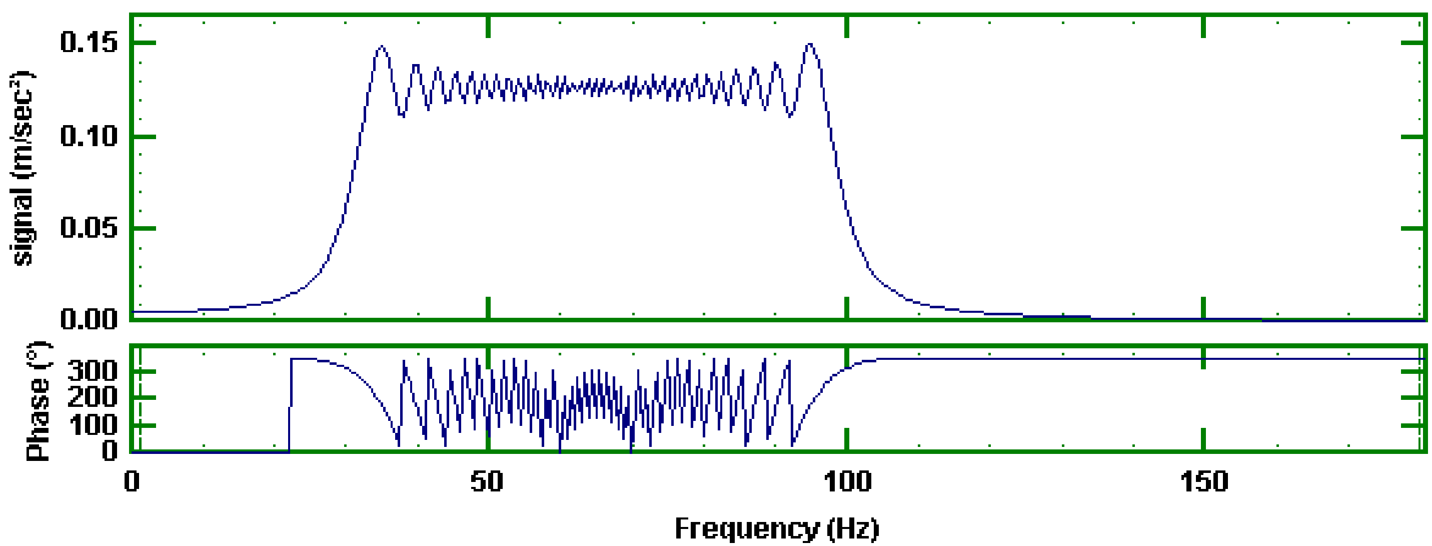
DIFFERENT TYPES OF EXCITATION: SLOW SWEEP SINE



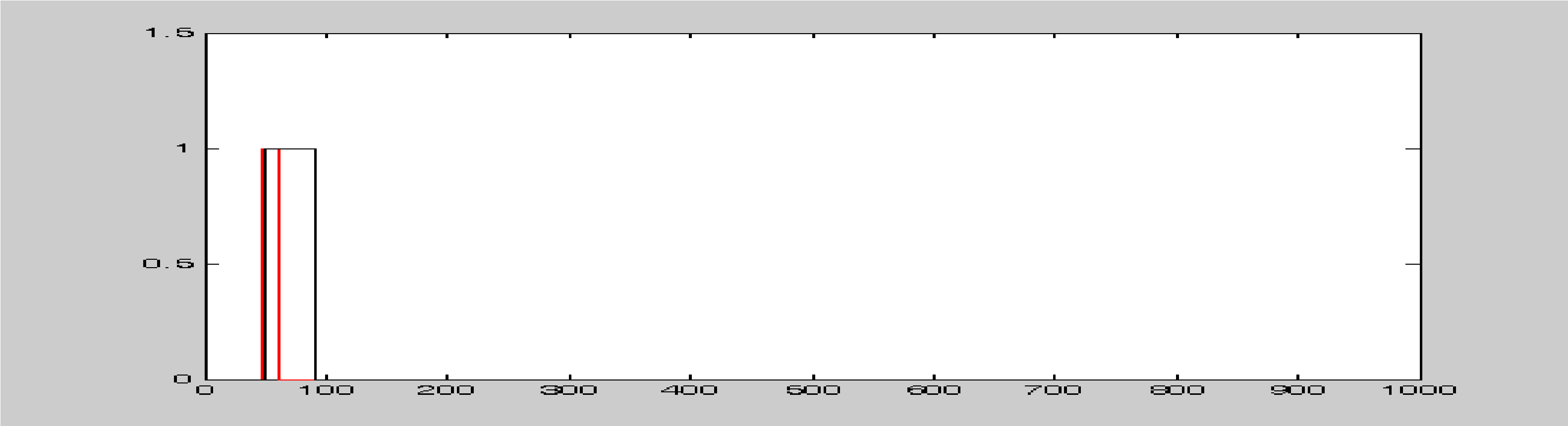


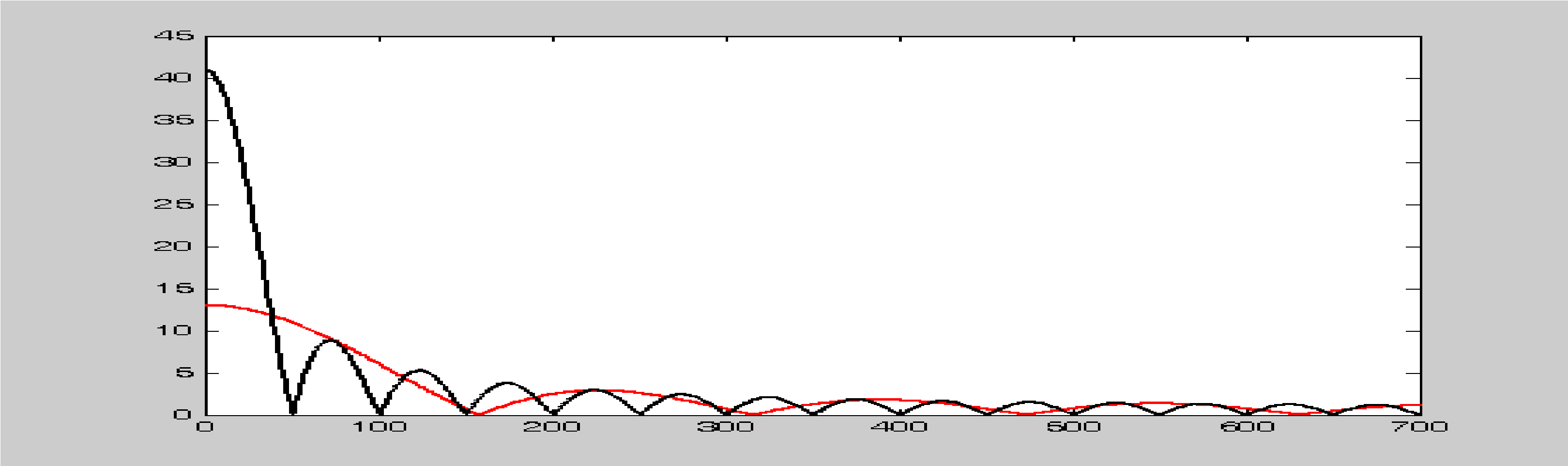
DIFFERENT TYPES OF EXCITATION: FAST SWEEP



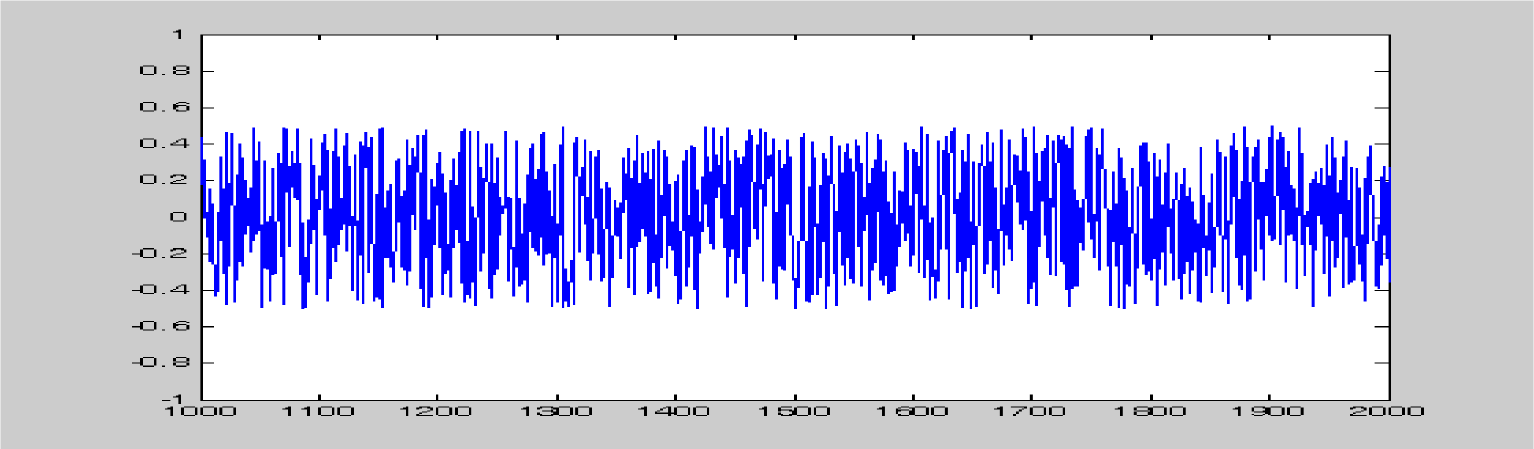


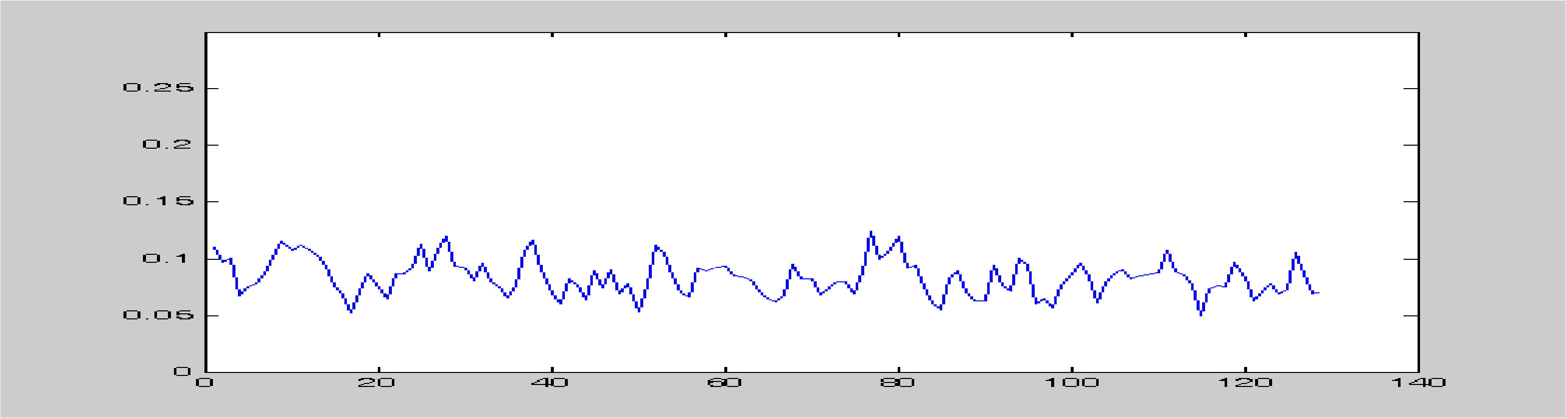
DIFFERENT TYPES OF EXCITATION: IMPULSE



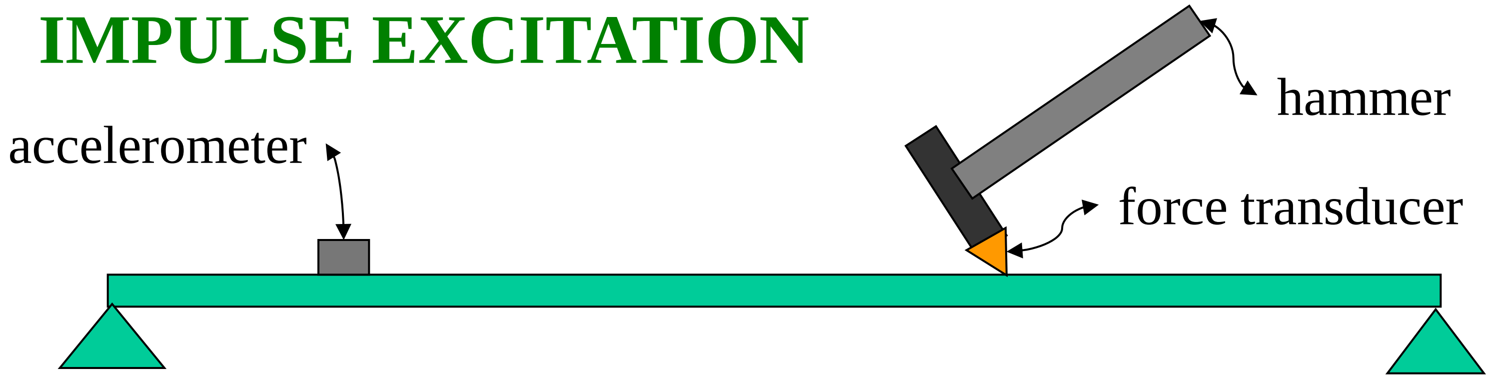


DIFFERENT TYPES OF EXCITATION: RANDOM





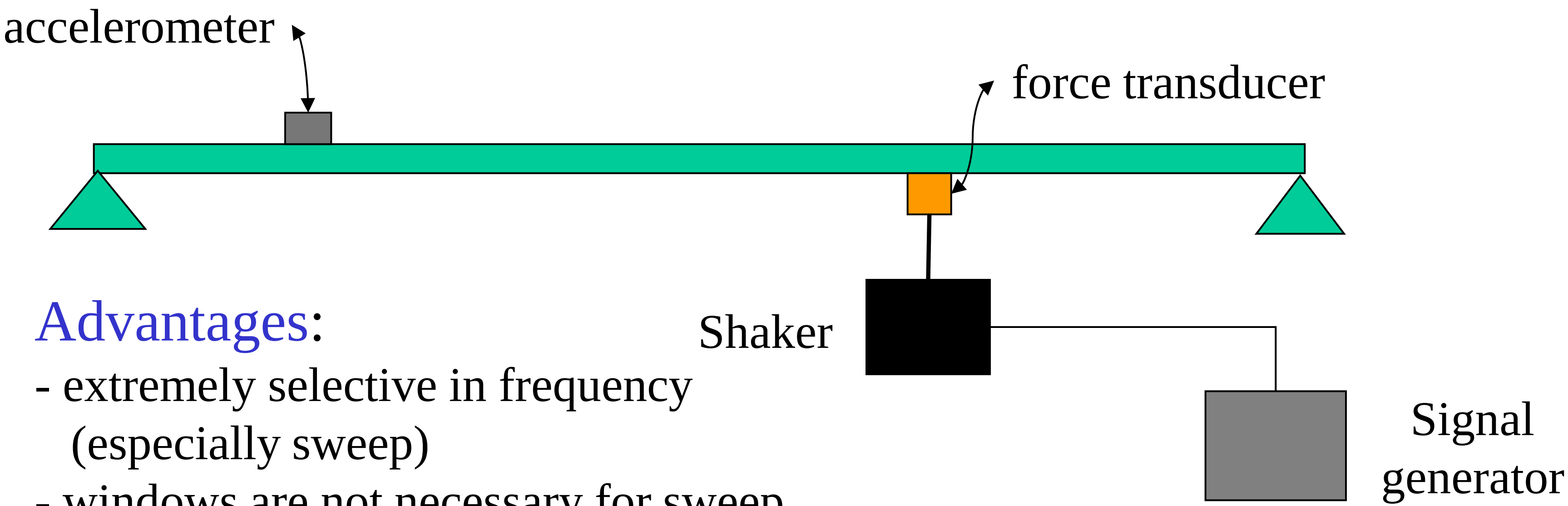
IMPULSE EXCITATION



Advantages:
- fast and inexpensive
- non invasive
- damping is not affected by the excitation

Disadvantages:
- is not frequency selective
- impulse duration is difficult to control
- input and output windows are necessary (transient for the input, exponential for the output)

FAST SWEEP AND RANDOM EXCITATIONS



Advantages:

- extremely selective in frequency (especially sweep)
- windows are not necessary for sweep, necessary (Hanning) for random
- very effective and precise

Disadvantages:

- invasive excitation
- difficult to locate appropriately the shaker
- damping is affected by the excitation

MULTI-DIRECTION MEASUREMENTS

When it is necessary to measure the response along three different axes, tri-axial accelerometers are generally used.

Open problems in modal testing are, on the contrary:
- the measurement of rotational response
- the application and measurement of moment excitation

These two measurements are necessary especially when trying to determine the FRF of coupled components from the FRF of the single components

DATA COLLECTION AND PARAMETERS ASSESSMENT

When preparing a test, the following parameters must be accurately estimated in view of the subsequent modal parameters identification:

- frequency range of interest: this choice depends on the number of modes that must be identified. Impulse testing can be not adequate when broad frequency bandwidths must be measured. Random or sweep excitations must be used;
- frequency resolution: this choice depends on how close are the natural frequencies of the component under test, recalling that for a correct identification of the modal parameters a sufficient number of *sampled points* are necessary;
- dimension of the FRF matrix (n. of response points to measure): this choice depends, as the first one, on the number of modes that must be identified. If only the natural frequencies are required, even a single response point can be sufficient

INDUSTRIAL MODAL ANALYSIS: ISSUES AND CHALLANGES

- Optimizing the test process
- Large structures (> 1000 points, in operating vehicles…)
- Novel transducers (MEMS, new lasers …)
- Wireless
- Complex structures, novel materials, high and distributed damping (uneven energy distribution)
- Multiple excitation (MIMO Tests)
- Use of a priori information for experiment design
- Nonlinearity checks, non-linear model detection and identification
- Excitation Design: Get maximal information in minimal time

SUMMARY AND OUTLOOK

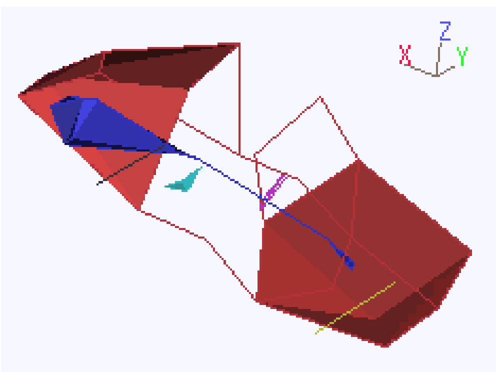
- Early product optimization is essential to meet market demands
- Mechanical Design Analysis and Optimization heavily rely on Structural Models
- Experimental Modal Analysis is the key approach, it is a de-facto standard in many industries
- While EMA is in essence a system identification problem, particular test and analysis issues arise due to model size and complexity
- Important challenges are related to supporting the industrial demands (test time and accuracy) and novel applications

CONCLUDING REMARKS

- Modal testing is an experimental process devoted to determine a vibration model aimed at identifying the modal parameters of a structure or a structural component
- It can carried out as input-output technique or output-only
- SISO, SIMO and MIMO techniques can be developed
- Different types of excitation can be produced: sequential excitation (stepped-sine or slow sweep sine) or transient excitation (impulse, fast sweep sine, random)

MODAL ANALYSIS PRINCIPLE: DECOMPOSITION IN EIGENMODES

Modal Analysis: The modal superposition



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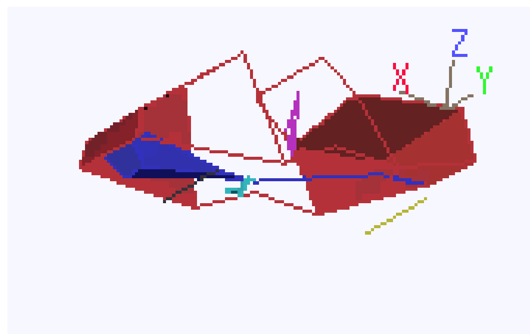

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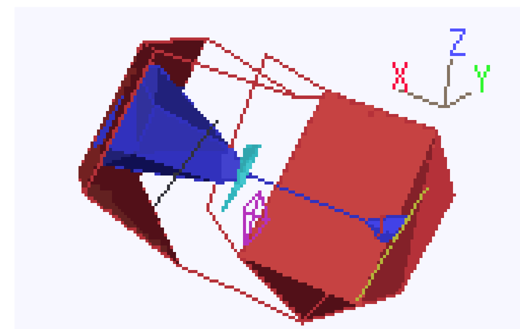

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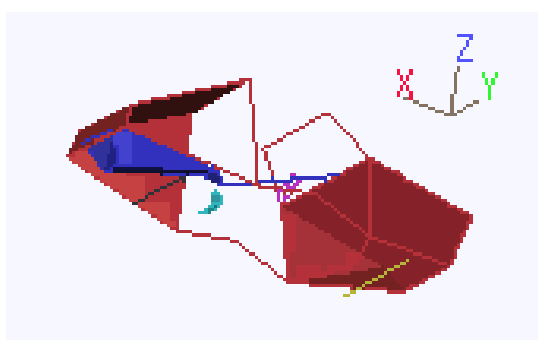

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